ADDENDUM FOR "LOCALLY CONSTRAINED INVERSE CURVATURE FLOWS"

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After publication of [1] we found out that the proof of Theorem 1.1 [1, p. 6798] is not complete, as the subsequential limiting spheres obtained from the equality case in the Heintze-Karcher inequality do not have to be centered at the origin.

In this addendum we provide the following workaround: Along the variation

$$\partial_t x = \left(\frac{H_{k-1}}{H_k} - \frac{u}{\lambda'}\right)\nu$$

the quermassintegrals W_k of the enclosed convex bodies evolve by

$$\partial_t W_k = c_{n,k}\int_{M_t}\frac{H_{k-1}\lambda' - uH_k}{\lambda'} = c_{n,k}\int_{M_t}\frac{H_k^{ij}\Lambda_{;ij}}{\lambda'},$$

where $\Lambda'(r) = \lambda(r)$ and hence

$$\Lambda_{;ij} = (\lambda r_{;i})_{;j} = \lambda' r_{;i}r_{;j} + \lambda r_{;ij} = \lambda' g_{ij} - uh_{ij}$$

and the positive constant $c_{n,k}$ may vary between the equalities. As H_k^{ij} is divergence free in spaceforms, integration by parts gives

$$\partial_t W_k = c_{n,k}\int_{M_t}\frac{\lambda''\lambda}{\lambda'^2}H_k^{ij}r_{;j}r_{;i} \leq -\delta\int_{M_t}H_k^{ij}r_{;i}r_{;j}$$

for some uniform constant δ, since in [1, Sec. 6,7] we have proven uniform barrier estimates as well as preservation of convexity. There can not exist $\epsilon > 0$, such that

$$\int_{M_t}H_k^{ij}r_{;i}r_{;j} \geq \epsilon \quad \forall t > 0,$$

for otherwise the long-time existence would imply that W_k turns negative in finite time, which is impossible. Hence there exists a subsequence of times (t_m) with

$$\int_{M_{t_m}}H_k^{ij}r_{;i}r_{;j} \to 0 \quad \text{as} \quad m \to \infty.$$

However, from [1, (8.4)] we know that any convergent subsequence of (M_{t_m}) must converge to a geodesic sphere. Pick such a subsequence (not relabelled), then along this subsequence H_k^{ij} converges to g^{ij} and we obtain that the limit M_{t_∞} satisfies

$$\int_{M_{t_\infty}}\|\nabla r\|^2 = 0.$$

Hence this limit is a sphere centered at the origin. Due to the barrier estimates, which imply arbitrary annuli are preserved by the flow, the whole flow must converge to a uniquely determined sphere centered at the origin.

References

[1] Julian Scheuer and Chao Xia, *Locally constrained inverse curvature flows*, Trans. Am. Math. Soc. **372** (2019), no. 10, 6771–6803, doi:10.1090/tran/7949 .