

**ADDENDUM FOR “QUANTITATIVE OSCILLATION ESTIMATES  
FOR ALMOST-UMBILICAL CLOSED HYPERSURFACES IN  
EUCLIDEAN SPACE”**

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We want to make an additional comment on [2, Thm. 1.1]. Unfortunately only after the publication of this paper we learned about the paper [1], which implies our result using standard results and a simple calculation. However, the methods of the proofs differ tremendously. In [1] the author uses projection methods onto 3-dimensional subspaces and standard pinching results for surfaces, whereas we use estimates in terms of  $L^p$  pinchings.

In fact, with our method it is also possible to prove closeness estimates in terms of  $\|\mathring{A}\|_p$ , namely that for  $p > n$  and a closed and strictly convex hypersurface with  $|M| = 1$  we find constants  $\beta = \beta(p, n)$  and  $\epsilon_0 = \epsilon_0(n, p, \|A\|_p)$ , such that if  $\epsilon < \epsilon_0$  and

$$\|\mathring{A}\|_p \leq \|H\|_p \epsilon,$$

then

$$\text{dist}(M, S_R) \leq c\epsilon^\beta.$$

Here one also has to be careful with the proposed exponent  $\beta = \frac{1}{2+\alpha}$ .  $\beta$  will in general become smaller when  $p$  gets closer to  $n$ . If we are not dealing with the case  $p = \infty$ , in which we would even obtain  $\beta = 1$  due to [1], we are not aware how  $\beta$  behaves in dependence of  $p > n$ .

REFERENCES

- [1] Kurt Leichtweiß, *Nearly umbilical ovaloids in the  $n$ -space are close to spheres*, Result. Math. **36** (1999), no. 1-2, 102–109.
- [2] Julian Scheuer, *Quantitative oscillation estimates for almost-umbilical closed hypersurfaces in Euclidean space*, Bull. Aust. Math. Soc. **92** (2015), no. 1, 133–144.